

# Extension and Application of Flux-Vector Splitting to Calculations on Dynamic Meshes

W. Kyle Anderson,\* James L. Thomas,\* and Christopher L. Rumsey\*  
NASA Langley Research Center, Hampton, Virginia

## Abstract

**T**HE Van Leer method of flux-vector splitting for the Euler equations is extended for use on moving meshes and all properties of the original splittings are maintained. The solution is advanced in time with an implicit, approximately factored algorithm. A subiterative procedure to minimize factorization and linearization errors so that larger time steps can be used is investigated. Subsequent computations are shown for a transonic wing undergoing forced pitching oscillation.

## Contents

### Computational Procedure

The governing equations are the time-dependent equations of ideal-gas dynamics, i.e., the Euler equations, which express the conservation of mass, momentum, and energy for an inviscid gas. The equations are written in generalized coordinates and conservation form as

$$\frac{\partial \hat{Q}}{\partial t} + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} + \frac{\partial \hat{H}}{\partial \zeta} = 0 \quad (1)$$

where  $\hat{Q}$  is the vector of conserved variables and  $\hat{F}$ ,  $\hat{G}$ , and  $\hat{H}$  represent the fluxes of mass, momentum, and energy in each of the spatial directions. The equations have been generalized from Cartesian coordinates using a transformation of the type

$$\begin{aligned} \xi &= \xi(x, y, z, t), & \eta &= \eta(x, y, z, t) \\ \zeta &= \zeta(x, y, z, t), & \tau &= t \end{aligned} \quad (2)$$

The equations are solved with an implicit, finite-volume, upwind-differenced algorithm in which the spatial derivatives of the fluxes are split into forward and backward contributions using flux-vector splitting so that type-dependent differencing can be used. The flux-vector splitting method used is that of Van Leer,<sup>1</sup> which has been widely used for flow computations on stationary meshes. It is continuously differentiable at eigenvalue sign changes and allows shocks to be captured with at most two interior zones. In practice, only one zone is usually observed. The details of the flux derivation for use on unsteady meshes can be found in Ref. 2. An alternate derivation has been subsequently given in Ref. 3.

Presented as Paper 87-1152 at the AIAA 8th Computational Fluid Dynamics Conference, Honolulu, HI, June 9-11, 1987; received Sept. 8, 1987; revision received May 25, 1988. Copyright © 1988 American Institute of Aeronautics and Astronautics, Inc. No copyright is asserted in the United States under Title 17, U.S. Code. The U.S. Government has a royalty-free license to exercise all rights under the copyright claimed herein for Governmental purposes. All other rights are reserved by the copyright owner. Full paper available at AIAA Library, 555 W. 57th St., New York, NY 10019. Price: microfiche, \$4.00; hard copy, \$9.00. Remittance must accompany order.

\*Research Scientist, Analytical Methods Branch, Low-Speed Aerodynamics Division. Member AIAA.

The resulting flux vectors maintain all the properties of the original splitting. The generalized flux  $\hat{F}$  for  $|M_\xi| < 1$  is

$$\hat{F}^\pm = \frac{|\text{grad}(\xi)|}{J} \begin{bmatrix} f_{\text{mass}}^\pm \\ f_{\text{mass}}^\pm \left[ \xi_x \frac{(-\bar{U} \pm 2a)}{\gamma} + u \right] \\ f_{\text{mass}}^\pm \left[ \xi_y \frac{(-\bar{U} \pm 2a)}{\gamma} + v \right] \\ f_{\text{mass}}^\pm \left[ \xi_z \frac{(-\bar{U} \pm 2a)}{\gamma} + w \right] \\ f_{\text{energy}}^\pm \end{bmatrix} \quad (3)$$

where

$$\begin{aligned} f_{\text{mass}}^\pm &= \pm \rho a (M_\xi \pm 1)^2 / 4 \\ f_{\text{energy}}^\pm &= f_{\text{mass}}^\pm \left\{ \left[ \frac{-(\gamma - 1)\bar{U}^2 \pm 2(\gamma - 1)\bar{U}a + 2a^2}{(\gamma^2 - 1)} \right] \right. \\ &\quad \left. + \frac{(u^2 + v^2 + w^2)}{2} - \frac{\xi_t}{\gamma} (-\bar{U} \pm 2a) \right\} \\ \bar{U} &= \xi_x u + \xi_y v + \xi_z w + \xi_t \\ (\xi_x, \xi_y, \xi_z, \xi_t) &= \frac{(\xi_x, \xi_y, \xi_z, \xi_t)}{|\text{grad}(\xi)|} \end{aligned}$$

Here,  $M_\xi = \bar{U}/a$  is the Mach number in the direction normal to a  $\xi = \text{constant}$  face and  $J$  the Jacobian of the transformation. The generalized fluxes in the other coordinate directions are easily obtained by substituting the appropriate metrics in place of  $\xi$ .

The solution is obtained with a spatially split approximate factorization algorithm that is second-order accurate in both time and space. The three-dimensional algorithm is implemented in three steps in which successive iterations can be used at each time level in order to minimize factorization and linearization errors.<sup>2</sup> The convergence rate of this process can be accelerated using a multigrid procedure as in Ref. 2. Generally, the convergence of the iterative process is very rapid so that only one or two cycles are sufficient. Larger time steps can be taken using the subiterative process than with the noniterative single step algorithm, which uses the result of the first iteration to approximate the solution at the new time. The additional work required by the subiterations at each time step partially offsets the advantage of taking the larger time step, therefore making the benefits of the subiterations marginal. The subiterative algorithm may prove to be more beneficial for operators that are incomplete linearizations in time, when higher time accuracy is used, or for calculations using the Navier-Stokes equations.

## Results

Unsteady results are compared with experiment<sup>4</sup> in Fig. 1 for the F-5 wing at a freestream Mach number of 0.95 under-

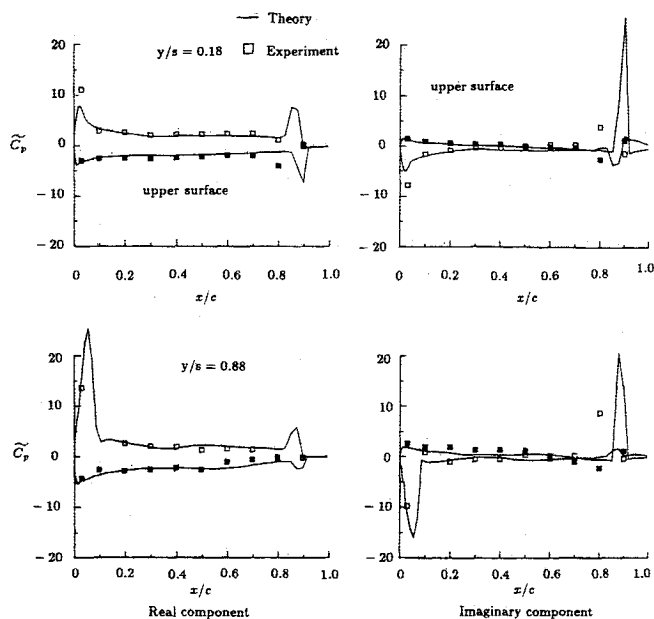


Fig. 1 Unsteady pressures for F-5 wing:  $M_\infty = 0.95$ ,  $k = 0.264$ ,  $\alpha_0 = 0$  deg, and  $\alpha_1 = 0.532$  deg.

going forced pitching motion given by

$$\alpha(\tau) = \alpha_0 + \alpha_1 \sin(M_\infty k \tau) \quad (4)$$

where  $k = 0.264$  is the reduced frequency based on root chord and  $\alpha_0 = 0$  deg,  $\alpha_1 = 0.532$  deg.

These results have been obtained on a  $129 \times 33 \times 33$  C-H mesh, corresponding to 129 points along the airfoil and wake, 33 points approximately normal to the airfoil, and 33 points in the spanwise direction, 17 of which are on the wing planform. A time step of 0.05 was used corresponding to approximately 250 time steps required per pitching cycle. Figure 1 shows the real and imaginary components of the pressure coefficient compared with experiment at two span stations:  $y/s = 0.18$  and 0.88. The results show reasonable comparison with experiment. The pressure spike at the shock, however, is somewhat aft of the experimental results. Although not shown, the shock location computed at this Mach number for steady flow at zero angle of attack is also aft of the data, indicating that viscous effects are important.

### References

- <sup>1</sup>Anderson, W. K., Thomas, J. L., and Van Leer, B., "A Comparison of Finite Volume Flux Vector Splittings for the Euler Equations," *AIAA Journal*, Vol. 24, Sept. 1986, pp. 1453-1460.
- <sup>2</sup>Anderson, W. K., Thomas, J. L., and Rumsey, C. L., "Extension and Applications of Flux-Vector Splitting to Unsteady Calculations on Dynamic Meshes," AIAA Paper 87-1152, June 1987.
- <sup>3</sup>Parpia, I. H., "Van Leer Flux Vector Splitting in Moving Coordinates," *AIAA Journal*, Vol. 26, Jan. 1988, pp. 113-115.
- <sup>4</sup>Tijedeman, H., Van Nunen, J. W. G., Kraan, A. N., Persoon, A. S., Rpestkoke, R., Roos, R., Schippers, P., and Siebert, C. M. "Transonic Wind Tunnel Test on an Oscillating Wing with External Stores," Air Force Flight Dynamics Lab., Wright-Patterson AFB, OH, AFFDL-TR-78-194, Dec. 1978.

Recommended Reading from the AIAA  
Progress in Astronautics and Aeronautics Series . . .



## Thermal Design of Aeroassisted Orbital Transfer Vehicles

H. F. Nelson, editor

Underscoring the importance of sound thermophysical knowledge in spacecraft design, this volume emphasizes effective use of numerical analysis and presents recent advances and current thinking about the design of aeroassisted orbital transfer vehicles (AOTVs). Its 22 chapters cover flow field analysis, trajectories (including impact of atmospheric uncertainties and viscous interaction effects), thermal protection, and surface effects such as temperature-dependent reaction rate expressions for oxygen recombination; surface-ship equations for low-Reynolds-number multicomponent air flow, rate chemistry in flight regimes, and noncatalytic surfaces for metallic heat shields.

TO ORDER: Write AIAA Order Department,  
370 L'Enfant Promenade, S.W., Washington, DC 20024  
Please include postage and handling fee of \$4.50 with all orders.  
California and D.C. residents must add 6% sales tax. All orders under  
\$50.00 must be prepaid. All foreign orders must be prepaid. Please allow  
4-6 weeks for delivery. Prices are subject to change without notice.

1985 566 pp., illus. Hardback  
ISBN 0-915928-94-9  
AIAA Members \$49.95  
Nonmembers \$74.95  
Order Number V-96