Extension and Application of Flux-Vector Splitting to Calculations on Dynamic Meshes

W. Kyle Anderson,* James L. Thomas,* and Christopher L. Rumsey*
NASA Langley Research Center, Hampton, Virginia

Abstract

THE Van Leer method of flux-vector splitting for the Euler equations is extended for use on moving meshes and all properties of the original splittings are maintained. The solution is advanced in time with an implicit, approximately factored algorithm. A subiterative procedure to minimize factorization and linearization errors so that larger time steps can be used is investigated. Subsequent computations are shown for a transonic wing undergoing forced pitching oscillation.

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Computational Procedure

The governing equations are the time-dependent equations of ideal-gas dynamics, i.e., the Euler equations, which express the conservation of mass, momentum, and energy for an inviscid gas. The equations are written in generalized coordinates and conservation form as

$$\frac{\partial \hat{Q}}{\partial t} + \frac{\partial \hat{F}}{\partial \xi} + \frac{\partial \hat{G}}{\partial \eta} + \frac{\partial \hat{H}}{\partial \zeta} = 0 \tag{1}$$

where $\hat{Q}$ is the vector of conserved variables and $\hat{F}$, $\hat{G}$, and $\hat{H}$ represent the fluxes of mass, momentum, and energy in each of the spatial directions. The equations have been generalized from Cartesian coordinates using a transformation of the type

$$\xi = \xi(x,y,z,t), \qquad \eta = \eta(x,y,z,t)$$
$$\zeta = \zeta(x,y,z,t), \qquad \tau = t \tag{2}$$

The equations are solved with an implicit, finite-volume, upwind-differenced algorithm in which the spatial derivatives of the fluxes are split into forward and backward contributions using flux-vector splitting so that type-dependent differencing can be used. The flux-vector splitting method used is that of Van Leer,[1] which has been widely used for flow computations on stationary meshes. It is continuously differentiable at eigenvalue sign changes and allows shocks to be captured with at most two interior zones. In practice, only one zone is usually observed. The details of the flux derivation for use on unsteady meshes can be found in Ref. 2. An alternate derivation has been subsequently given in Ref. 3.

The resulting flux vectors maintain all the properties of the original splitting. The generalized flux $\hat{F}$ for $|M_\xi| < 1$ is

$$\hat{F}^{\pm} = \frac{|\mathrm{grad}(\xi)|}{J} \begin{bmatrix} f^{\pm}_{\mathrm{mass}} \\ f^{\pm}_{\mathrm{mass}} \left[\hat{\xi}_x \dfrac{(-\bar{U} \pm 2a)}{\gamma} + u \right] \\ f^{\pm}_{\mathrm{mass}} \left[\hat{\xi}_y \dfrac{(-\bar{U} \pm 2a)}{\gamma} + v \right] \\ f^{\pm}_{\mathrm{mass}} \left[\hat{\xi}_z \dfrac{(-\bar{U} \pm 2a)}{\gamma} + w \right] \\ f^{\pm}_{\mathrm{energy}} \end{bmatrix} \tag{3}$$

where

$$f^{\pm}_{\mathrm{mass}} = \pm \rho a (M_\xi \pm 1)^2/4$$

$$f^{\pm}_{\mathrm{energy}} = f^{\pm}_{\mathrm{mass}} \left\{ \left[\frac{-(\gamma-1)\bar{U}^2 \pm 2(\gamma-1)\bar{U}a + 2a^2}{(\gamma^2-1)} \right] + \frac{(u^2+v^2+w^2)}{2} - \frac{\hat{\xi}_t}{\gamma}(-\bar{U} \pm 2a) \right\}$$

$$\bar{U} = \hat{\xi}_x u + \hat{\xi}_y v + \hat{\xi}_z w + \hat{\xi}_t$$

$$(\hat{\xi}_x, \hat{\xi}_y, \hat{\xi}_z, \hat{\xi}_t) = \frac{(\xi_x, \xi_y, \xi_z, \xi_t)}{|\mathrm{grad}(\xi)|}$$

Here, $M_\xi = \bar{U}/a$ is the Mach number in the direction normal to a ξ = constant face and J the Jacobian of the transformation. The generalized fluxes in the other coordinate directions are easily obtained by substituting the appropriate metrics in place of ξ.

The solution is obtained with a spatially split approximate factorization algorithm that is second-order accurate in both time and space. The three-dimensional algorithm is implemented in three steps in which successive iterations can be used at each time level in order to minimize factorization and linearization errors.[2] The convergence rate of this process can be accelerated using a multigrid procedure as in Ref. 2. Generally, the convergence of the iterative process is very rapid so that only one or two cycles are sufficient. Larger time steps can be taken using the subiterative process than with the noniterative single step algorithm, which uses the result of the first iteration to approximate the solution at the new time. The additional work required by the subiterations at each time step partially offsets the advantage of taking the larger time step, therefore making the benefits of the subiterations marginal. The subiterative algorithm may prove to be more beneficial for operators that are incomplete linearizations in time, when higher time accuracy is used, or for calculations using the Navier-Stokes equations.

Results

Unsteady results are compared with experiment[4] in Fig. 1 for the F-5 wing at a freestream Mach number of 0.95 under-

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*Research Scientist, Analytical Methods Branch, Low-Speed Aerodynamics Division. Member AIAA.

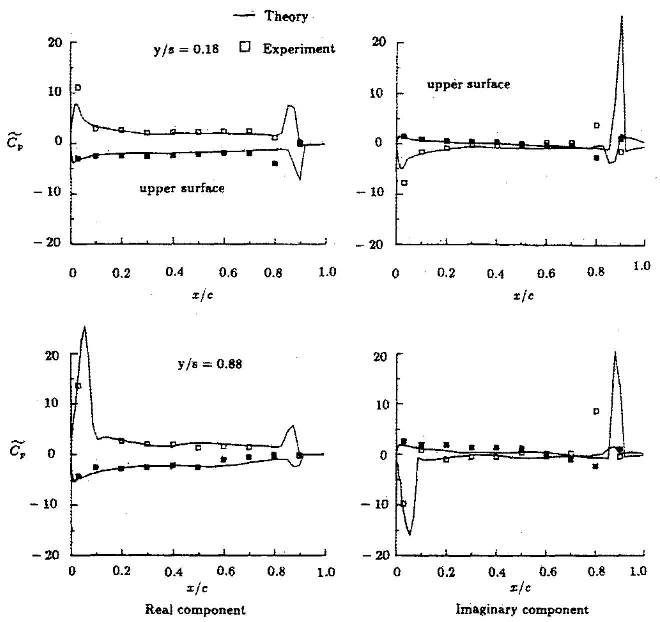

Fig. 1 Unsteady pressures for F-5 wing: $M_\infty = 0.95$, $k = 0.264$, $\alpha_0 = 0$ deg, and $\alpha_1 = 0.532$ deg.

going forced pitching motion given by

$$\alpha(\tau) = \alpha_0 + \alpha_1 \sin(M_\infty k \tau) \tag{4}$$

where $k = 0.264$ is the reduced frequency based on root chord and $\alpha_0 = 0$ deg, $\alpha_1 = 0.532$ deg.

These results have been obtained on a $129 \times 33 \times 33$ C-H mesh, corresponding to 129 points along the airfoil and wake, 33 points approximately normal to the airfoil, and 33 points in the spanwise direction, 17 of which are on the wing planform. A time step of 0.05 was used corresponding to approximately 250 time steps required per pitching cycle. Figure 1 shows the real and imaginary components of the pressure coefficient compared with experiment at two span stations: $y/s = 0.18$ and 0.88. The results show reasonable comparison with experiment. The pressure spike at the shock, however, is somewhat aft of the experimental results. Although not shown, the shock location computed at this Mach number for steady flow at zero angle of attack is also aft of the data, indicating that viscous effects are important.

References

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